Asymmetric Volatility — GJR-GARCH

Week 4 — Financial Management: Volatility, Risk, and AI

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Today's Roadmap

1. The Story: Why This Matters
2. The GJR-GARCH Model
3. The News Impact Curve
4. EGARCH: An Alternative
5. Python Live Demo
6. Application: COVID Crash
7. Key Takeaways

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The Story: Why This Matters

The VolTech Challenge

Last week: Alex built GARCH(1,1) models.

But the standardized residuals showed negative skewness: after a -3% day, tomorrow's volatility was significantly higher than after a $+3\%$ day.

GARCH uses r_{t-1}^2 — the square **erases the sign**.

The Core Problem

Bad news hits harder than good news. GARCH is blind to this asymmetry.

Today's Learning Objectives

By the end of this session, you will:

1. Explain the leverage effect and its economic mechanism
2. Master the GJR-GARCH model and its γ parameter
3. Read and construct the News Impact Curve
4. Compare GJR-GARCH with EGARCH
5. Conduct a likelihood ratio test for asymmetry

Two Theories of Asymmetric Volatility

1. **Leverage Effect** (Black, 1976):
   Price falls $\rightarrow$ debt/equity rises $\rightarrow$ higher financial risk $\rightarrow$ higher volatility
2. **Volatility Feedback** (Campbell & Hentschel, 1992):
   Expected vol rises $\rightarrow$ required return rises $\rightarrow$ price drops immediately

⚠ Both predict the same pattern

Negative returns $\Rightarrow$ disproportionately larger volatility increase. This is universal across equity markets.

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The GJR-GARCH Model

GJR-GARCH(1,1): Breaking the Symmetry

GJR-GARCH(1,1) (Glosten, Jagannathan & Runkle, 1993)

$$\sigma_t^2 = \omega + \alpha\, r_{t-1}^2 + \gamma\, r_{t-1}^2\, \mathbf{1}(r_{t-1} < 0) + \beta\, \sigma_{t-1}^2$$

- $\mathbf{1}(r_{t-1} < 0)$: **indicator function** — equals 1 for bad news, 0 for good news
- $\gamma > 0$: negative shocks get **extra weight**
- When $\gamma = 0$: reduces to standard GARCH

How Asymmetry Works

Shock Type	GARCH	GJR-GARCH
Positive ($r_{t-1} > 0$)	$\alpha \cdot r_{t-1}^2$	$\alpha \cdot r_{t-1}^2$
Negative ($r_{t-1} < 0$)	$\alpha \cdot r_{t-1}^2$	$(\alpha + \gamma) \cdot r_{t-1}^2$
Asymmetry ratio	1.0	$\frac{\alpha+\gamma}{\alpha}$

Typical equities: $\alpha \approx 0.02$–0.05, $\gamma \approx 0.10$–0.15

Asymmetry ratio ≈ 3–$5\times$: negative shocks have 3–5 times the impact of positive shocks!

Stationarity and Long-Run Variance

GJR-GARCH Stationarity Condition

$$\alpha + \frac{\gamma}{2} + \beta < 1$$

The $\gamma/2$ arises because the indicator is active $\approx 50\%$ of the time (symmetric distribution).

Long-Run Variance

$$\bar{\sigma}^2 = \frac{\omega}{1 - \alpha - \gamma/2 - \beta}$$

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The News Impact Curve

NIC: Visualizing Asymmetry

News Impact Curve

$$\text{NIC}(r_{t-1}) = \omega + \alpha\, r_{t-1}^2 + \gamma\, r_{t-1}^2\, \mathbf{1}(r_{t-1} < 0) + \beta\, \bar{\sigma}^2$$

GARCH NIC:

- Symmetric parabola
- Same curvature left and right

GJR-GARCH NIC:

- Asymmetric parabola
- Steeper on left (bad news)
- **Kink** at $r_{t-1} = 0$

Key diagnostic: The NIC is the single best chart for showing regulators how the model responds to good vs. bad news.

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EGARCH: An Alternative

EGARCH(1,1) (Nelson, 1991)

EGARCH Variance Equation

$$\ln(\sigma_t^2) = \omega + \alpha\left(|z_{t-1}| - \mathbb{E}[|z_{t-1}|]\right) + \gamma\, z_{t-1} + \beta\, \ln(\sigma_{t-1}^2)$$

Key differences from GJR-GARCH:

Feature	GJR-GARCH	EGARCH
Models	σ_t^2	$\ln(\sigma_t^2)$
Positivity	Needs constraints	Always positive
Leverage sign	$\gamma > 0$	$\gamma < 0$
NIC shape	Piecewise quadratic	Exponential

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Python Live Demo

Step 1: Fit Three Models

```python
from arch import arch_model
import numpy as np, pandas as pd, yfinance as yf

sp500 = yf.download("^GSPC",
    start="2015-01-01", end="2024-12-31")
prices = sp500["Close"].squeeze()
returns = 100 * np.log(
    prices / prices.shift(1)).dropna()

# Standard GARCH
res_g = arch_model(returns, vol="Garch",
    p=1, q=1, dist="t").fit(disp="off")
# GJR-GARCH (o=1 adds asymmetry)
res_gjr = arch_model(returns, vol="Garch",
    p=1, o=1, q=1, dist="t").fit(disp="off")
# EGARCH
res_e = arch_model(returns, vol="EGARCH",
    p=1, o=1, q=1, dist="t").fit(disp="off")
```

Step 2: Interpret GJR Parameters

```python
alpha = res_gjr.params["alpha[1]"]
gamma = res_gjr.params["gamma[1]"]
beta  = res_gjr.params["beta[1]"]
omega = res_gjr.params["omega"]

print(f"alpha = {alpha:.4f}")
print(f"gamma = {gamma:.4f}")
print(f"beta  = {beta:.4f}")
print(f"Asymmetry ratio: "
      f"{(alpha+gamma)/alpha:.1f}x")

persist = alpha + gamma/2 + beta
longrun = np.sqrt(
    omega/(1-persist) * 252)
print(f"Long-run vol: {longrun:.2f}%")
```

Step 3: News Impact Curve

```python
import matplotlib.pyplot as plt
shock = np.linspace(-5, 5, 500)

# GARCH NIC (symmetric)
a_g = res_g.params["alpha[1]"]
b_g = res_g.params["beta[1]"]
o_g = res_g.params["omega"]
lrv_g = o_g / (1 - a_g - b_g)
nic_g = o_g + a_g*shock**2 + b_g*lrv_g

# GJR NIC (asymmetric)
lrv_j = omega/(1 - alpha - gamma/2 - beta)
nic_j = (omega + alpha*shock**2
    + gamma*shock**2*(shock<0)
    + beta*lrv_j)

plt.plot(shock, nic_g, label="GARCH")
plt.plot(shock, nic_j, label="GJR-GARCH")
plt.legend()
```

Step 4: Likelihood Ratio Test

```python
from scipy import stats

lr = 2 * (res_gjr.loglikelihood
           - res_g.loglikelihood)
p_val = 1 - stats.chi2.cdf(lr, df=1)

print(f"LR statistic: {lr:.2f}")
print(f"p-value: {p_val:.6f}")
print(f"Reject gamma=0 at 5%: "
      f"{'Yes' if p_val < 0.05 else 'No'}")

# Model comparison
for name, r in [("GARCH", res_g),
    ("GJR", res_gjr), ("EGARCH", res_e)]:
    print(f"{name:8s} AIC={r.aic:.1f} "
          f"BIC={r.bic:.1f}")
```

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Application: COVID Crash

GARCH vs. GJR-GARCH During COVID

COVID-19 crash (Feb–Apr 2020):

- Many consecutive negative return days
- Each day activated the γ term
- GJR-GARCH ramped up faster and higher

Quantitative gap:

- GARCH peak vol: $\sim$78%
- GJR peak vol: $\sim$94%
- Difference: 16 pct points

Practical Impact

Using GARCH instead of GJR-GARCH after a market crash **systematically understates risk** — when accurate risk measurement matters most.

Model Diagnostics: Residual Improvement

After GJR-GARCH fitting, check standardized residuals:

Diagnostic	**GARCH**	**GJR-GARCH**
Ljung–Box p (sq. resid)	> 0.05	> 0.05
Residual skewness	-0.31	≈ -0.15
Residual excess kurtosis	≈ 2.0	≈ 1.8

- Both remove autocorrelation in squared residuals
- GJR-GARCH **reduces skewness** by capturing asymmetry
- Residual kurtosis remains $> 0 \Rightarrow$ Student-t distribution still needed

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Key Takeaways

Six Things to Remember

1. **Leverage effect**: Negative returns increase future volatility more than positive returns (Black, 1976)
2. **GJR-GARCH**: Adds $\gamma\, r_{t-1}^2\, \mathbf{1}(r_{t-1} < 0)$ — one parameter captures the asymmetry
3. **News Impact Curve**: The key diagnostic tool. Symmetric parabola (GARCH) vs. kinked parabola (GJR)
4. **EGARCH**: Models $\ln(\sigma_t^2)$, always positive. Same economics, different math ($\gamma < 0 =$ leverage)
5. **LR test**: Formally tests $\gamma = 0$. For equities, asymmetry is almost always significant
6. **Practical impact**: Ignoring asymmetry $=$ underestimating risk after market declines

Mission 4: The Asymmetry Report

Deliverables

1. Fit GARCH, GJR-GARCH, EGARCH on Nikkei & S&P 500
2. Report parameters, AIC, BIC, log-likelihood
3. Conduct LR test: GARCH vs. GJR-GARCH
4. Plot News Impact Curves (all 3 models)
5. COVID volatility comparison (GARCH vs. GJR)
6. One-page model recommendation memo to Kenji

Next Week Preview

Week 5: Volatility Forecasting

David asks the hard question: “Can this model *predict* tomorrow’s risk? Not just fit yesterday’s data?”

Priya runs an out-of-sample horse race: GARCH vs. GJR-GARCH vs. EGARCH vs. her LSTM.

Topics: Out-of-sample evaluation, loss functions (MSE, QLIKE), Mincer–Zarnowitz regression, forecast combination